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CONSTITUTIVE MODELING AND THERMOVISCOPLASTICITY

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Development and solution of coupled thermomechanical equations at elevated temperature and/or high strain rates are discussed. Three main considerations are presented: development of the coupled thermomechanical equations by means of the rational theory of thermodynamics, development of a thermoviscoplastic constitutive equation which is congruous with the developed coupled equations, and the applicability of the developed equations to the treatment by the finite element method.

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Introduction

It is well established that finite deformations of a solid body particularly at elevated temperature and/or high strain rates represent coupled thermomechanical processes, which require the simultaneous solution of the coupled balance of momenta and energy equations. A proper development and solution of such thermomechanical problems requires: 1) adoption of the rational theory of thermodynamics, 2) a comprehensive viscoplastic constitutive equation which accounts for the strain rate, temperature and hardening effects, and 3) compatibility with the available numerical tools, particularly the finite element method. These requirements taken together have not been used extensively by researchers in dealing with the coupled thermomechanical problems. However, because of the need for stringent accuracy when solving practical thermomechanical problems such as in rockets and in nuclear reactors, the importance of these requirements is being recognized. Inoue and Nagaki [1] and Allen [2] developed coupled thermomechanical equations with limited applications to one dimensional problems. Ghoneim [3] presented a coupled equations, without hardening effects, and applied them to a two dimensional axisymmetric problem of compression of a constrained-ends cylinder. Lehmann [4] presented a comprehensive analysis of the development of the coupled equations with application to the necking problem in a specimen subjected to the tensile test. However, a more realistic constitutive law which includes strain rate and temperature effects is needed.

In this paper development and solution of the coupled thermomechanical problems is considered, based on the three requirements listed earlier: The coupled thermomechanical equations are developed on the basis of the rational theory of thermodynamics; a viscoplastic constitute equation which accounts for the temperature, strain rate, and hardening effect is proposed; and the computability of the developed coupled thermomechanical equation with the finite element method is discussed.

Development of the Coupled Thermomechanical Equations

We assume that the state of a material point is oompletely determined by the knowledge of the elastic strain tensor $\underline{\underline{E}}^e$, the inelastic strain tensor $\underline{\underline{E}}^I$, the absolute temperature T, the temperature gradient $\underline{\nabla} T$, and a set of internal state variables $\underline{\underline{\alpha}}^i$, i = 1,..., p. Consequently, the following constitutive relations may be postulated:

$$\underline{\underline{S}} = \underline{\underline{S}} \left(\underline{\underline{E}}^e, \underline{\underline{E}}^I, T, \underline{\nabla T}, \underline{\underline{\alpha}}^i \right) \tag{1.1}$$

$$\psi = \psi \left(\underline{\underline{E}}^e, \underline{\underline{E}}^I, T, \underline{\nabla T}, \underline{\underline{\alpha}}^i \right) \tag{1.2}$$

$$s = s \left(\underline{\underline{E}}^e, \underline{\underline{E}}^I, T, \underline{\nabla T}, \underline{\underline{\alpha}}^i \right) \tag{1.3}$$

$$\underline{q} = \underline{q} \left(\underline{\underline{E}}^e, \underline{\underline{E}}^I, T, \underline{\nabla T}, \underline{\underline{\alpha}}^i \right) \tag{1.4}$$

$$\dot{\underline{\underline{E}}}^I = \underline{\underline{g}}_1 \left(\underline{\underline{E}}^e, \underline{\underline{E}}^I, T, \underline{\nabla T}, \underline{\underline{\alpha}}^i \right) \tag{1.5}$$

and

$$\dot{\underline{\underline{\alpha}}}^i = \underline{\underline{g}}_2 \left(\underline{\underline{E}}^e, \underline{\underline{E}}^I, T, \underline{\nabla T}, \underline{\underline{\alpha}}^i \right) \tag{1.6}$$

where $\underline{\underline{S}}$ is the second Piola-Kirchhoff stress tensor, ψ stands for the Helmholtz free energy, s means the specific entropy, and q is the heat flux per unit area.

Upon invoking the axiom of admissibility (i.e., the compatibility of the assumed constitutive relations with the fundamental equations of mechanics) and when adopting the separabilty of the total Green-Lagrange strain energy

$$\underline{\underline{E}} = \underline{\underline{E}}^e + \underline{\underline{E}}^I + \underline{\underline{\gamma}} T$$

where $\underline{\underline{\gamma}}$ is the coefficient of the thermal expansion tensor, it follows:

1) $\psi = \psi\,(\underline{\underline{E}}^e, \underline{\underline{E}}^I, T, \underline{\underline{\alpha}}^i)$

$$s = \frac{\partial\psi}{\partial\underline{\underline{E}}^e} : \underline{\underline{\gamma}} - \frac{\partial\psi}{\partial T} = s\,(\underline{\underline{E}}^e, \underline{\underline{E}}^I, T, \underline{\underline{\alpha}}^i)$$

$$\underline{\underline{S}} = \rho\,\frac{\partial\psi}{\partial\underline{\underline{E}}^e} = \underline{\underline{S}}\,(\underline{\underline{E}}^e, \underline{\underline{E}}^I, T, \underline{\underline{\alpha}}^i)$$

2) $$\rho\,\left(\frac{\partial\psi}{\partial\underline{\underline{E}}^I} : \dot{\underline{\underline{E}}}^I + \frac{\partial\psi}{\partial\underline{\underline{\alpha}}} : \dot{\underline{\underline{\alpha}}} + \dot{s}T\right) = \underline{\underline{S}} : \underline{\underline{E}}^I - \underline{\nabla}\cdot\underline{q} \qquad \ldots(2)$$

and 3) $$\left(\underline{\underline{S}} - \rho\,\frac{\partial\psi}{\partial\underline{\underline{E}}^e}\right) : \dot{\underline{\underline{E}}}^I + \rho\,\frac{\partial\psi}{\partial\underline{\underline{\alpha}}} : \dot{\underline{\underline{\alpha}}} - \underline{q}\,\frac{\nabla\theta}{\theta} \geq 0 \qquad \ldots(3)$$

At this stage, we may postulate

$$\rho\psi = \rho\psi_0 + \tfrac{1}{2}\,\underline{\underline{E}}^e : D^4 : \underline{\underline{E}}^e + \rho\,c_v\,T\,(1 - \ln T) + \underline{\underline{\beta}} : \underline{\underline{\alpha}} \qquad \ldots(4)$$

where D^4 is the fourth order elasticity tensor, ρ is the density at the reference configuration, and c_v stands for the specific heat at constant deformation. The tensor $\underline{\underline{\beta}}$ is the material property tensor. Substituting (4) and the Fourier's law, $\underline{q} = -k\underline{\nabla}T$ into equations (2) and (3), we obtain

$$-k\nabla^2 T + \rho c_v \dot{T} = -\rho(\underline{\underline{\gamma}} : D^4 : \dot{\underline{\underline{E}}}^e)\,T + \underline{\underline{S}} : \dot{\underline{\underline{E}}}^I - \rho\,\underline{\underline{\beta}} : \dot{\underline{\underline{\alpha}}} \qquad \ldots(5)$$

and

$$\underline{\underline{S}} : \dot{\underline{\underline{E}}}^I + \rho\,\underline{\underline{\beta}} : \dot{\underline{\underline{\alpha}}} + k\,\frac{(\nabla T)^2}{T} \geq 0$$

Equation (5) is the coupled heat equation which together with the balance of linear momentum equation constitutes the set of coupled thermomechanical equations. It might be worth pointing out that the right-hand side of the equation (5) represents the mechanical energy generation; the first term stands for the reversible part, the second for the dissipated irreversible part, and the third for the stored irreversible part due to microstructural effects.

The Viscoplactic Constitutive Equation

Solution of the coupled thermomechanical equations requires closed-form expressions for (1.5) and (1.6). These may be taken from any of the state variable theories [5-8]. Inhere, a constitutive equation which may be regarded as a modification of th Bodner-Partom's power law is proposed. The proposed equations are thought to be simple, lucid, and consequently very practical. As in the case of the internal state variables theory, the proposed constitutive equations utilize two state variables: a kinematic hardening state variable which accounts for the "rest" stress, and an isotropic hardening state variable which accounts for the "drag" stress. Only the isotropic hardening state variable will be considered in this paper.

After adopting the flow rule, we can show that

$$\dot{\underline{\underline{E}}}^{I} = \gamma_\circ \left(\frac{\tau^E}{Y} \right)^n \frac{\underline{\underline{S}}'}{\tau^E} \qquad \ldots(6)$$

where τ^E is the effective stress $\left(\tau^E = \sqrt{\frac{3}{2} S'_{ij} S'_{ij}} \right)$, $\underline{\underline{S}}'$ is the deviatoric stress tensor, n is a strain rate sensitivity parameter, and Y is the isotropic hardening state variable which is equivalent to the dynamic yield stress [8]. In general, Y is a functional of the history of deformation or any related quantity such as the viscoplastic work W^P. If the convolution form of Stieltjes integral is adopted for such functional

$$Y = \int_0^t e^{-\tau/\tau_0} \, W^p \,(t - \tau)\, d\tau$$

where t is the time and τ_0 stands for the relaxation time constant, and when the 3-parameter element model is considered, we get

$$\dot{Y} + aY = H_1 \dot{W}^p + H_2 \sqrt{W^p} \qquad \ldots(7)$$

where a, H_1, and H_2 are material constants.

In order to incorporaté the temperature effects into the evolution equations (6) and (7), Y is expressed as a function of temperature. Since in the proposed constitutive equations Y can be viewed as the equivalent dynamic yield stress, the function may be constructed from experimental data of the yield stress versus temperature. A possible form of this function is

$$Y = Y_o \left(\frac{Tc - T}{Tc - To}\right) \qquad \ldots(8)$$

where T_c is a constant, T_o is a reference temperature, and Y_o is the value of the yield stress at T_o. In addition, from the observation of the variation of the plastic flow with temperature, we have $\dot{\underline{\underline{E}}}^I$ proportional to exp (-Q/RT),

$$\dot{\underline{\underline{E}}} \sim \exp(-\ Q/RT) \qquad (9)$$

where Q is the activation energy (assumed constant), and R is the universal gas constant. From equations (6), (8) and (9) it follows that n must be a function of T,

$$n = \frac{Q}{R}\left(\frac{T-To}{ToT}\right) / \ln\left(\frac{Tc-To}{Tc-T}\right) \qquad \ldots(10)$$

The proposed viscoplastic constitutive equations (equations (6) and (7) subjected to (8) and (10)) are examined by conducting a series of one-dimensional uniaxial numerical calculations. Samples of the results are given in Figures 1-4. Figures 1 and 2 display the tensile stress-strain results at different strain rates and temperature. Strain rate history effects are demonstrated by a jump test in Figure 3. Cyclic test results, Figure 4, depict the cyclic hardening effects. Qualitatively speaking, results demonstrate the capability of the proposed viscoplastic constitutive equations in generating some of the important characteristics of a class of viscoplastic materials. A quantitative investigation of the constitutive equations is to be conducted experimentally for some viscoplastic materials in a future work.

Finite Element Implementation

Compatibility of the developed coupled thermomechanical equations is demonstrated for the case of quasistatic infinitesimal deformation with no body force and no heat generation, i.e.,

$$\underline{\nabla}\cdot\underline{\underline{\sigma}} = 0 \qquad \ldots(11)$$

$$-k\nabla^2 T + \rho c_v \dot{T} = \underline{\sigma}\cdot\underline{\dot{\varepsilon}}^p - \left(\frac{E}{1-2\nu}\right) \dot{\varepsilon}_v T - \rho\beta\dot{\gamma} \qquad \ldots(12)$$

where $\underline{\sigma} = [D]\ (\underline{\varepsilon} - \underline{\varepsilon}^{vp} - \gamma\ T\underline{\delta})$

and $\underline{\varepsilon}^{vp} = \gamma_0\ (\frac{\tau^E}{Y})\ \frac{\underline{\sigma}^1}{\tau^E}$

γ_0 is a scalara constant, ρ is the density, $\underline{\delta}$ stands for the Kronecker symbol, $\underline{\sigma}$ and $\underline{\varepsilon}$ are the stress and strain tensors, respectively, expressed in vector form, $\underline{\sigma}^1$ and $\underline{\varepsilon}^{vp}$ the corresponding deviatoric stress and viscoplastic strain vectors, respectively, ε_v is the dilatation, and [D] is the elastic matrix.

When adopting the Galerkin finite element method, (11) and (12) become, respectively,

$$[K1]\ \underline{\dot{U}} + [C1]\ \underline{\dot{T}} = \underline{R} + \underline{F}_1 \qquad \ldots(13)$$

$$[C2]\ \underline{\dot{T}} + [K2]\ \underline{T} = \underline{Q} + \underline{F}_2 \qquad \ldots(14)$$

where $\underline{U}$, $\underline{T}$, $\underline{R}$, and $\underline{Q}$ are the nodal displacement, the nodal temperature, the nodal force, and the thermal convection load vectors, respectively, $\underline{F}_1$ is a vector which accounts for the viscoplastic effects of the balance of momentum equation, and $\underline{F}_2$ is a vector which accounts for the mechanical heat generation. Also, [K1], [K2], [C1], and [C2] are, respectively, the stiffness, conductivity, coupling, and consistency matrices. The differential equations (13) and (14) can be solved by using the general "θ" method in conjunction with the fixed point iteration method for the solution of the ensuing nonlinear algebraic equations. Results of tensile and compression loading of a constrained-ends cylinder for a constant Y can be found in [8].

The extension of this work to incorporate hardening and temperature effects and solving other practical problems is being undertaken.

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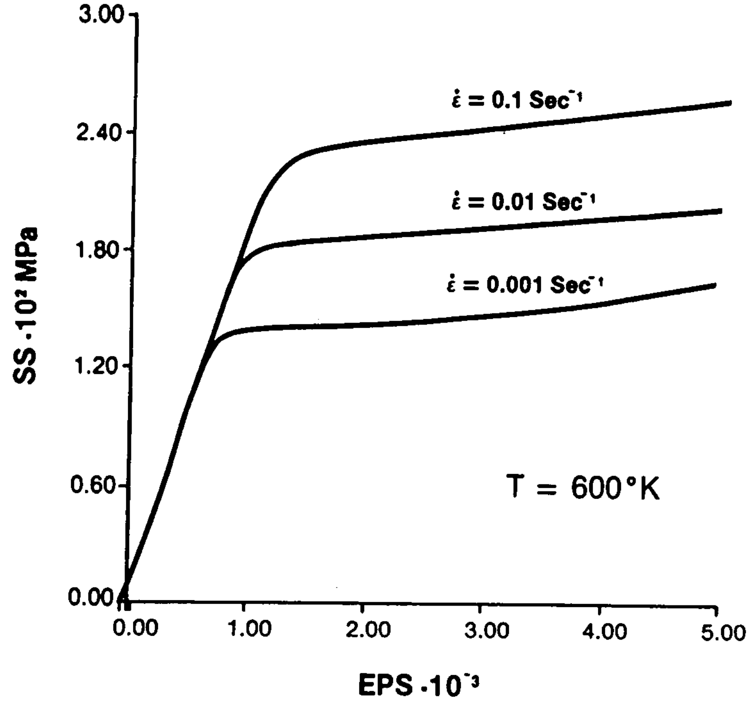
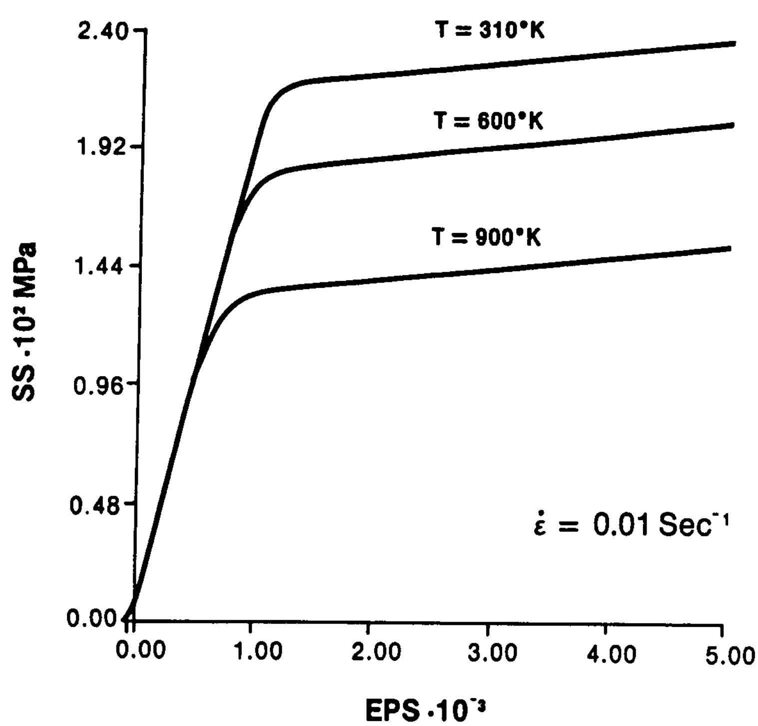


$\dot{\varepsilon}$ = 0.1 Sec^{-1}
$\dot{\varepsilon}$ = 0.01 Sec^{-1}
$\dot{\varepsilon}$ = 0.001 Sec^{-1}
T = 600°K
SS·10² MPa
3.00
2.40
1.80
1.20
0.60
0.00
0.00
1.00
2.00
3.00
4.00
5.00
EPS·10⁻³

Fig. 1



T = 310°K
T = 600°K
T = 900°K
$\dot{\varepsilon}$ = 0.01 Sec^{-1}
SS·10² MPa
2.40
1.92
1.44
0.96
0.48
0.00
0.00
1.00
2.00
3.00
4.00
5.00
EPS·10⁻³

Fig. 2

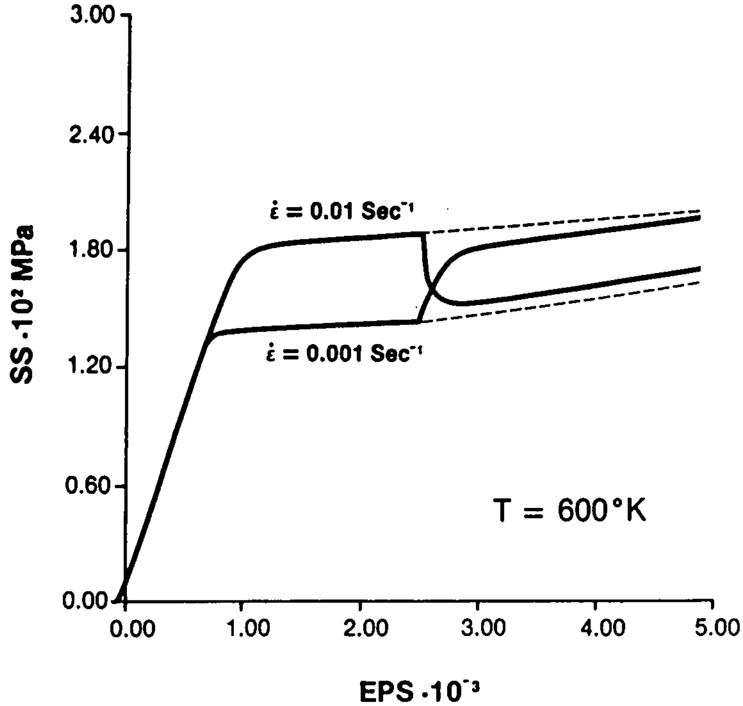

3.00
2.40
1.80
1.20
0.60
0.00
SS ·10² MPa
$\dot{\varepsilon}$ = 0.01 Sec⁻¹
$\dot{\varepsilon}$ = 0.001 Sec⁻¹
T = 600°K
0.00
1.00
2.00
3.00
4.00
5.00
EPS ·10⁻³

Fig. 3

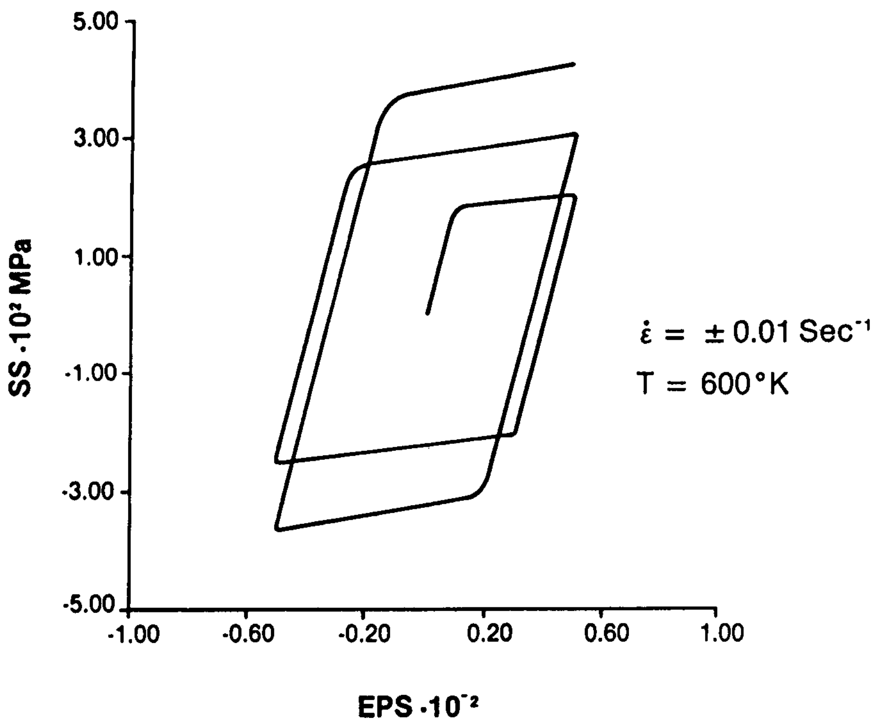

5.00
3.00
1.00
-1.00
-3.00
-5.00
SS ·10² MPa
$\dot{\varepsilon}$ = ± 0.01 Sec⁻¹
T = 600°K
-1.00
-0.60
-0.20
0.20
0.60
1.00
EPS ·10⁻²

Fig. 4